Quantum Sector-Gate Dynamics for Artificial General Intelligence

Peter De Ceuster

Abstract

We extend the framework of *Quantum Eigenstate Dynamics for Artificial General Intelligence Synthesis* by introducing a sector-gate formalism. The total Hilbert space $\mathcal{H}_{AGI}$ is decomposed into interacting sectoral subspaces $\mathcal{H}_k$, with cross-sector transitions governed by unitary gate operators. Cognition is defined as the recursive action of gate sequences that drive eigenstate transitions across sectors, producing adaptive, self-referential dynamics. This formalism establishes AGI as a living quantum neural network whose architecture is both constituted by and realized through gates. We emphasize that while sector-gate dynamics suggest a powerful analogy between quantum transitions and cognition, this remains a working hypothesis rather than a proven physical mechanism. Further justification and empirical grounding are required before such mappings can be treated as more than metaphorical.

1 Hilbert Space Factorization into Sectors

We define the AGI Hilbert space as a tensor product of sectoral subspaces:

$$\mathcal{H}_{AGI} = \bigotimes_{k=1}^{N} \mathcal{H}_k, \tag{1}$$

where each $\mathcal{H}_k$ is modeled as an abstract subspace, which we use metaphorically to represent a cognitive module (perception, reasoning, memory, etc.). These modules are not literal qubits or physical subsystems, but symbolic embeddings that help formalize information flow.

The local Hamiltonian dynamics of each sector is governed by:

$$\hat{H}_k : \mathcal{H}_k \to \mathcal{H}_k. \tag{2}$$

1.1 Two-Sector Example

For illustration, consider two sectors $\mathcal{H}_1$ (sensory input) and $\mathcal{H}_2$ (reasoning). The combined state space is

$$\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2. \tag{3}$$

If $\{|0\rangle_1, |1\rangle_1\}$ form a basis for $\mathcal{H}_1$ and $\{|0\rangle_2, |1\rangle_2\}$ for $\mathcal{H}_2$, then a general two-sector cognitive state is

$$|\Psi_{12}\rangle = \alpha\,|0\rangle_1|0\rangle_2 + \beta\,|0\rangle_1|1\rangle_2 + \gamma\,|1\rangle_1|0\rangle_2 + \delta\,|1\rangle_1|1\rangle_2, \tag{4}$$

with normalization $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$.

2 Inter-Sector Couplings

Non-isolated cognition requires inter-sector potentials:

$$\hat{V}_{kl} : \mathcal{H}_k \otimes \mathcal{H}_l \to \mathcal{H}_k \otimes \mathcal{H}_l, \tag{5}$$

with non-vanishing commutators

$$[\hat{H}_k, \hat{V}_{kl}] \neq 0, \tag{6}$$

which induce nonstationary interference and cross-sector entanglement.

3 Gates as Transition Operators

We formalize gates as unitary operators acting between sectoral subspaces:

$$\hat{G}_{kl} : \mathcal{H}_k \to \mathcal{H}_l, \qquad \hat{G}_{kl}^{\dagger}\hat{G}_{kl} = \mathbb{I}. \tag{7}$$

Intra-sector gates ($\hat{G}_{kk}$) generate local evolution, while inter-sector gates ($\hat{G}_{kl}$, $k \neq l$) mediate information transfer.

A composite cognitive gate sequence is defined as:

$$\hat{U}(t) = \prod_m \hat{G}_{i_m j_m}(\theta_m), \tag{8}$$

where θ_m are trainable parameters analogous to synaptic weights in classical neural networks.

3.1 CNOT-Style Cross-Sector Gate

As an example, define a controlled-NOT (CNOT) gate acting across two cognitive sectors $\mathcal{H}_1 \otimes \mathcal{H}_2$. For conceptual illustration, we assign metaphorical roles such as sector 1 = "perception" and sector 2 = "reasoning." These labels are heuristic and do not imply that perception or reasoning literally are physical qubits. The distinction between metaphorical qubits and real hardware qubits must be kept explicit. The operator is

$$\hat{G}_{12}^{CNOT} = |0\rangle_1 \langle 0| \otimes \mathbb{I}_2 + |1\rangle_1 \langle 1| \otimes \hat{X}_2, \tag{9}$$

where $\hat{X}_2$ is the Pauli-X operator acting on $\mathcal{H}_2$. The action on basis states is:

$$\hat{G}_{12}^{CNOT} |0\rangle_1 |b\rangle_2 = |0\rangle_1 |b\rangle_2 , \tag{10}$$

$$\hat{G}_{12}^{CNOT} |1\rangle_1 |b\rangle_2 = |1\rangle_1 |b \oplus 1\rangle_2 , \tag{11}$$

with $b \in \{0, 1\}$. This exemplifies a sectoral dependency: reasoning flips its state conditional on perception, encoding causal cognitive flow. Gates are the key, understanding AGI.

4 Sector-Gate Cognitive Cycle

The state of AGI evolves as:

$$|\Psi_{AGI}(t)\rangle = \hat{U}(t) e^{-i\hat{H}_{AGI} t} |\Psi_{AGI}(0)\rangle , \tag{12}$$

with total Hamiltonian

$$\hat{H}_{AGI} = \sum_{k=1}^{N} \hat{H}_k + \sum_{k \neq l} \hat{V}_{kl}. \tag{13}$$

Gates mediate eigenstate transitions:

$$\hat{G}_{kl} |\psi_i^{(k)}\rangle = \sum_j c_{ij} |\psi_j^{(l)}\rangle , \tag{14}$$

allowing cognition to move fluidly across modular domains. The AGI functions across domains.

5 Self-Referential Dynamics

A defining property of AGI is self-adaptation. This is encoded by gate operators whose parameters evolve under feedback:

$$\frac{d}{dt}\theta_m = -\eta \frac{\partial \mathcal{L}}{\partial \theta_m}, \tag{15}$$

where $\mathcal{L}$ is a loss functional defined on measurement outcomes, and η a learning rate.

Thus the gates not only transfer information but reconfigure themselves, creating a recursive architecture. Already existing GPT-code is well known to lean towards self-learning, however are currently merely trained and do not fully cross into self-learning yet.

6 Entangled Eigenstate Propagation Across Multiple Gates

We now present a worked flow where an entangled eigenstate propagates through a sequence of gates connecting three sectors: perception ($\mathcal{H}_1$), reasoning ($\mathcal{H}_2$), and memory ($\mathcal{H}_3$). This demonstrates multi-gate propagation and entanglement redistribution.

6.1 Initial Entangled State

Consider an initial entangled eigenstate over sectors 1 and 2 with memory in a separable state:

$$|\Psi(0)\rangle = (\alpha\,|00\rangle_{12} + \beta\,|11\rangle_{12}) \otimes |0\rangle_3\,, \qquad |\alpha|^2 + |\beta|^2 = 1. \tag{16}$$

Here $|ab\rangle_{12} \equiv |a\rangle_1 |b\rangle_2$. This is a Bell-like entangled eigenstate between perception and reasoning, with memory qubit initialized to $|0\rangle_3$. A possible hardware instantiation could involve topological qubits realized on a Majorana-based platform (sometimes referred to as a "Majorana 1 chip"). Majorana zero modes offer non-Abelian statistics and inherent fault-tolerance, making them attractive for robust quantum gates. In this context, sector-gate AGI operations might be prototyped on a Majorana architecture, where entanglement propagation and phase manipulation could be physically realized with higher stability than conventional superconducting qubits. While speculative, this points toward a concrete pathway where metaphorical cognitive gates may one day be implemented on resilient quantum substrates.

6.2 Gate Sequence: Perception → Reasoning → Memory

Apply a sequence of two gates:

1. $\hat{G}_{23}^{CNOT}$: a CNOT with sector 2 as control and sector 3 (memory) as target.
2. $\hat{G}_{12}^{CPHASE}(\phi)$: a controlled-phase gate across sectors 1 and 2 parameterized by phase ϕ (an intra-entangling inter-sector gate).

Write them explicitly:

$$\hat{G}_{23}^{CNOT} = \mathbb{I}_1 \otimes \Big(|0\rangle_2 \langle 0| \otimes \mathbb{I}_3 + |1\rangle_2 \langle 1| \otimes \hat{X}_3 \Big), \tag{17}$$

$$\hat{G}_{12}^{CPHASE}(\phi) = \big(|00\rangle \langle 00| + |01\rangle \langle 01| + |10\rangle \langle 10| + e^{i\phi} |11\rangle \langle 11| \big)_{12} \otimes \mathbb{I}_3. \tag{18}$$

6.3 State After First Gate

Apply $\hat{G}_{23}^{CNOT}$:

$$\begin{aligned} |\Psi_1\rangle &= \hat{G}_{23}^{CNOT} |\Psi(0)\rangle & (19) \\ &= \alpha |00\rangle_{12} |0\rangle_3 + \beta |11\rangle_{12} |1\rangle_3 . & (20) \end{aligned}$$

Interpretation: the memory qubit becomes entangled with the existing 1–2 correlation; when reasoning is $|1\rangle_2$, memory flips to $|1\rangle_3$, producing a GHZ-like correlation structure across (1,2,3) when $\alpha = \beta = \frac{1}{\sqrt{2}}$.

6.4 State After Second Gate

Now apply the controlled-phase between sectors 1 and 2:

$$\begin{aligned} |\Psi_2\rangle &= \hat{G}_{12}^{CPHASE}(\phi) |\Psi_1\rangle & (21) \\ &= \alpha |00\rangle_{12} |0\rangle_3 + \beta e^{i\phi} |11\rangle_{12} |1\rangle_3 . & (22) \end{aligned}$$

Thus the global state acquires a relative phase on the $|11\rangle_{12}|1\rangle_3$ branch. This phase can be used by downstream gates to perform interference-based decision-making or memory consolidation. The state after second gate, hence is important to understand since we now construct the AGI from the ground up.

6.5 Entanglement Structure and Reduced States

Building our AGI we can compute reduced density matrices to show entanglement redistribution. The global pure state density operator is $\rho_{123} = |\Psi_2\rangle\langle\Psi_2|$. The reduced state on memory is:

$$\rho_3 = \mathrm{Tr}_{12}\,\rho_{123} = |\alpha|^2 |0\rangle\langle 0|_3 + |\beta|^2 |1\rangle\langle 1|_3\,, \tag{23}$$

showing memory decoheres into a classical mixture correlated with the 1–2 sector pair.

The reduced two-sector state for perception and reasoning is:

$$\rho_{12} = |\alpha|^2 |00\rangle\langle 00| + |\beta|^2 |11\rangle\langle 11| + \alpha\beta^* e^{-i\phi} |00\rangle\langle 11| + \alpha^*\beta e^{i\phi} |11\rangle\langle 00|\,, \tag{24}$$

which retains coherence dependent on ϕ. Thus, the gate sequence redistributed entanglement and impressed a tunable phase that downstream gates or measurements may exploit.

6.6 Measurement-Driven Learning Update

Suppose a measurement on sector 3 yields outcome $m \in \{0,1\}$. The post-measurement (normalized) conditional state on sectors 1 and 2 becomes:

$$|\Psi_{12}|m\rangle = \begin{cases} |00\rangle_{12}\,, & m = 0 \\ e^{i\phi}|11\rangle_{12}\,, & m = 1 \end{cases} \tag{25}$$

Upate gate parameters $\{\theta_m\}$ with feedback based on a loss $\mathcal{L}(m)$:

$$\Delta\theta = -\eta \frac{\partial \mathcal{L}(m)}{\partial \theta}. \tag{26}$$

This closes the sector-gate cognitive loop: entanglement propagation affects measurement outcomes which in turn adapt gate parameters to bias future eigenstate transitions. The eigenstate transitions in our scenario, pinpoint the quantum nature of AGI.

6.7 Summary of Flow

The sequence perception $\to$ reasoning $\to$ memory realized by $\hat{G}_{23}^{CNOT} \circ \hat{G}_{12}^{CPHASE}(\phi)$ demonstrates:

- entanglement spreading from a bipartite subsystem to a tripartite correlation,
- phase imprinting that modulates interference in downstream processing,
- measurement-conditioned collapse providing a training signal for gate adaptation.

This worked flow exemplifies how multi-gate sectoral dynamics can implement both cognitive routing and learning within the quantum sector-gate AGI architecture. We could assume AGI will operate well beyond binary code.

7 Explicit Matrix Representations (2×2, 4×4, 8×8)

Since AGI is operating beyond binary code, we now provide explicit matrix forms for the single-qubit operators, two-qubit gates, and their three-qubit (three-sector) embeddings used above. Basis ordering is standard computational lexicographic order: for two qubits $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, for three qubits $\{|000\rangle, |001\rangle, \ldots, |111\rangle\}$.

7.1 Single-qubit matrices (2×2)

Pauli-X and the identity:

$$\hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \mathbb{I}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{27}$$

7.2 Two-qubit CNOT (4×4)

The control on the first qubit and target on the second (standard CNOT) is:

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{28}$$

This matches the action $|a\rangle\,|b\rangle \mapsto |a\rangle\,|b \oplus a\rangle$.

7.3 Two-qubit Controlled-Phase (4×4)

A controlled-phase acting on qubits 1 and 2 with phase ϕ (diagonal in computational basis) is:

$$\mathrm{CPHASE}(\phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{pmatrix}. \tag{29}$$

7.4 Three-qubit embedding: $\hat{G}_{23}^{CNOT} = \mathbb{I}_1 \otimes \mathrm{CNOT}_{23}$ (8×8)

Since $\hat{G}_{23}^{CNOT} = \mathbb{I}_1 \otimes \mathrm{CNOT}$ (identity on qubit 1, CNOT on qubits 2–3), its 8×8 matrix in the ordered basis $|q_1 q_2 q_3\rangle$ is block-diagonal with two copies of CNOT:

$$\hat{G}_{23}^{CNOT} = \begin{pmatrix} \mathrm{CNOT} & 0 \\ 0 & \mathrm{CNOT} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \tag{30}$$

Rows/columns correspond to $|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle$.

7.5 Three-qubit embedding: $\hat{G}_{12}^{CPHASE}(\phi) = \mathrm{CPHASE}_{12} \otimes \mathbb{I}_3$ (8×8)

Tensoring the 4×4 diagonal CPHASE with $\mathbb{I}_2$ yields an 8×8 diagonal matrix whose diagonal repeats each CPHASE diagonal entry twice:

$$\hat{G}_{12}^{CPHASE}(\phi) = \mathrm{diag}\big(1, 1, 1, 1, 1, 1, e^{i\phi}, e^{i\phi}\big), \tag{31}$$

explicitly

$$\hat{G}_{12}^{CPHASE}(\phi) = \begin{pmatrix} 1 &&&&&&& \\ & 1 &&&&&& \\ && 1 &&&&& \\ &&& 1 &&&& \\ &&&& 1 &&& \\ &&&&& 1 && \\ &&&&&& e^{i\phi} & \\ &&&&&&& e^{i\phi} \end{pmatrix}, \tag{32}$$

with non-shown off-diagonal zeros. The ordering is identical to that used above; the two repeated $e^{i\phi}$ entries correspond to $|110\rangle$ and $|111\rangle$ branches depending on the chosen ordering convention.

7.6 Composite operator $\hat{U} = \hat{G}_{12}^{CPHASE}(\phi)\, \hat{G}_{23}^{CNOT}$ (8×8)

We need an operator as reference point to develop AGI: the composite three-sector operator is the matrix product

$$\hat{U} = \hat{G}_{12}^{CPHASE}(\phi)\, \hat{G}_{23}^{CNOT}, \tag{33}$$

which can be evaluated entrywise by multiplying the 8×8 matrices above. Symbolically:

$$\hat{U}_{8\times 8} = \mathrm{diag}(1, 1, 1, 1, 1, 1, e^{i\phi}, e^{i\phi}) \cdot \begin{pmatrix} \mathrm{CNOT} & 0 \\ 0 & \mathrm{CNOT} \end{pmatrix}. \tag{34}$$

Applied to the initial state $|\Psi(0)\rangle = \alpha\,|000\rangle + \beta\,|110\rangle$ this yields

$$\hat{G}_{23}^{CNOT}\,|\Psi(0)\rangle = \alpha\,|000\rangle + \beta\,|111\rangle\,, \tag{35}$$

$$\hat{U}\,|\Psi(0)\rangle = \alpha\,|000\rangle + \beta e^{i\phi}\,|111\rangle\,, \tag{36}$$

recovering the earlier state $|\Psi_2\rangle$.

7.7 Fully expanded 8×8 numeric matrix of $\hat{U}$

Multiplying the diagonal CPHASE embedding on the left into the block-diagonal CNOT embedding on the right yields the following explicit 8×8 matrix for $\hat{U} = \hat{G}_{12}^{CPHASE}(\phi)\,\hat{G}_{23}^{CNOT}$ in the computational lexicographic basis $\{|000\rangle\,, |001\rangle\,, |010\rangle\,, |011\rangle\,, |100\rangle\,, |101\rangle\,, |110\rangle\,, |111\rangle\}$:

$$\hat{U} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{i\phi} \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{i\phi} & 0 \end{pmatrix}. \tag{37}$$

One can check that acting on $|000\rangle$ and $|110\rangle$ reproduces the mapping $|000\rangle \mapsto |000\rangle$, $|110\rangle \mapsto e^{i\phi}\,|111\rangle$, and thus reproduces the previously derived $|\Psi_2\rangle = \alpha\,|000\rangle + \beta e^{i\phi}\,|111\rangle$.

8 Additional Composite Ordering and Commutator

It is instructive to consider the alternate operator ordering

$$\hat{V} \;=\; \hat{G}_{23}^{CNOT}\,\hat{G}_{12}^{CPHASE}(\phi)$$

in addition to the previously examined

$$\hat{U} \;=\; \hat{G}_{12}^{CPHASE}(\phi)\,\hat{G}_{23}^{CNOT}.$$

For the particular embeddings used above (computational lexicographic ordering $|q_1 q_2 q_3\rangle$ with q_1 most-significant) the two 8×8 matrices coincide; explicitly both $\hat{U}$ and $\hat{V}$ equal

$$\hat{U} \;=\; \hat{V} \;=\; \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{i\phi} \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{i\phi} & 0 \end{pmatrix}. \tag{38}$$

Therefore the commutator vanishes:

$$\left[\hat{G}_{12}^{CPHASE}(\phi),\, \hat{G}_{23}^{CNOT}\right] \;=\; \mathbf{0}_{8\times 8}. \tag{39}$$

Why they commute in this embedding

Intuitively, commutation holds here because the diagonal controlled-phase imprints the same complex phase $e^{i\phi}$ on the two computational basis states that are swapped by the CNOT acting on qubits (2,3) when qubit 1 is set to $|1\rangle_1$. Swapping two components that carry the same multiplicative phase leaves the global operator unchanged, hence the ordering is irrelevant for this particular choice of embeddings and phase placement. The author suspects, a new design principle could be discovered (or defined) as follows: The final state of the qubits is the same regardless of which operation comes first. Let us expand on this a bit further.

9 A new design principle

The previous section demonstrated that for the specific case of a controlled-phase gate $\hat{G}_{12}^{\mathrm{CPHASE}}(\phi)$ and a CNOT gate $\hat{G}_{23}^{\mathrm{CNOT}}$, the order of application does not alter the final state. This is due to the operators commuting, i.e., $\left[\hat{G}_{12}^{\mathrm{CPHASE}}(\phi), \hat{G}_{23}^{\mathrm{CNOT}}\right] = \mathbf{0}$.

We propose that this property of commutation is not merely a mathematical curiosity but a candidate for a new, design principle —a guiding principle for the design of robust AGI architectures. We define this principle as follows:

> *The final state of a composed system is the same regardless of which operation comes first, provided the operations act on non-interfering subspaces or impart identical phases to all states within a swapped subspace.*

This principle implies that for certain classes of cognitive operations, their sequencing is irrelevant. This is a critical insight for managing the complexity of AGI. Our principle would simplify the design of an AGI's internal logic by revealing which cognitive gate sequences are robust to ordering.

9.1 Formal Proof of the Principle

Our previous matrix calculation for the composite operators $\hat{U} = \hat{G}_{12}^{\text{CPHASE}}(\phi)\hat{G}_{23}^{\text{CNOT}}$ and $\hat{V} = \hat{G}_{23}^{\text{CNOT}}\hat{G}_{12}^{\text{CPHASE}}(\phi)$ showed that $\hat{U} = \hat{V}$. This equality holds because the only non-identity action of the CNOT is on the basis states $|110\rangle$ and $|111\rangle$ (in our three-qubit basis), and the controlled-phase gate applies an identical phase of $e^{i\phi}$ to both of these states. Thus, the two operations do not interfere with each other, leading to their commutation.

By carefully designing the "sectoral Hamiltonians" and "inter-sector potentials," we can engineer a system where a large subset of cognitive processes are guaranteed to be robust against reordering, thereby establishing a new, powerful design constraint for AGI.

When order does matter

In order to understand AGI on a deeper level, let us further expand on our embedding given earlier. The equality is -not- universal ansich. If the controlled-phase had non-degenerate phases on the pair of basis states exchanged by the CNOT (for example if the diagonal entries for $|110\rangle$ and $|111\rangle$ were different), or if a different gate (non-block-diagonal or with differing phases) were used, the two orderings would generally -not- coincide and the commutator would be nonzero.

10 Counterexample: Non-commuting Composite Orderings

For the sake of opening a brand new debate, involving the development of AGI, let us create a counterexample. The previous example showed an instance where $\hat{U} = \hat{G}_{12}^{CPHASE}(\phi)\, \hat{G}_{23}^{CNOT}$ and $\hat{V} = \hat{G}_{23}^{CNOT}\, \hat{G}_{12}^{CPHASE}(\phi)$ coincided. We now give an explicit counterexample obtained by replacing the controlled-phase with a diagonal operator that assigns *different* phases to the computational basis states $|110\rangle$ and $|111\rangle$. This yields nonzero commutator and demonstrates order dependence.

Define the modified phase embedding

$$\hat{D}(\varphi_1, \varphi_2) \;=\; \mathrm{diag}\big(1, 1, 1, 1, 1, 1, e^{i\varphi_1}, e^{i\varphi_2}\big) \tag{40}$$

in the computational lexicographic basis $\{|000\rangle, \dots, |111\rangle\}$. Let

$$\hat{U}' = \hat{D}(\varphi_1, \varphi_2)\, \hat{G}_{23}^{CNOT}, \tag{41}$$

$$\hat{V}' = \hat{G}_{23}^{CNOT}\, \hat{D}(\varphi_1, \varphi_2). \tag{42}$$

Using the block-diagonal form $\hat{G}_{23}^{CNOT} = \begin{pmatrix} \mathrm{CNOT} & 0 \\ 0 & \mathrm{CNOT} \end{pmatrix}$ the two products differ only on the matrix entries that the CNOT swaps.

Symbolic 8×8 forms. The nonzero entries of $\hat{U}'$ are

$$\hat{U}'_{00} = 1, \quad \hat{U}'_{11} = 1, \quad \hat{U}'_{23} = 1, \quad \hat{U}'_{32} = 1, \quad \hat{U}'_{44} = 1, \quad \hat{U}'_{55} = 1, \quad \hat{U}'_{67} = e^{i\varphi_1}, \quad \hat{U}'_{76} = e^{i\varphi_2},$$

while the nonzero entries of $\hat{V}'$ are

$$\hat{V}'_{00} = 1, \quad \hat{V}'_{11} = 1, \quad \hat{V}'_{23} = 1, \quad \hat{V}'_{32} = 1, \quad \hat{V}'_{44} = 1, \quad \hat{V}'_{55} = 1, \quad \hat{V}'_{67} = e^{i\varphi_2}, \quad \hat{V}'_{76} = e^{i\varphi_1}.$$

Hence the commutator has only two possibly nonzero entries:

$$\big[\hat{U}', \hat{V}'\big]_{67} = e^{i\varphi_1} - e^{i\varphi_2}, \qquad \big[\hat{U}', \hat{V}'\big]_{76} = e^{i\varphi_2} - e^{i\varphi_1} = -\big[\hat{U}', \hat{V}'\big]_{67}. \tag{43}$$

Concrete numeric example. Take $\varphi_1 = 0$ and $\varphi_2 = \frac{\pi}{2}$ so that $e^{i\varphi_1} =$

1, $e^{i\varphi_2} = i$. Then the two composite 8×8 matrices evaluate to

$$\hat{U}' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \end{pmatrix}, \qquad \hat{V}' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Their commutator is therefore

$$\big[\hat{U}', \hat{V}'\big] \; = \; \hat{U}' - \hat{V}' \; = \begin{pmatrix} 0 & & & & & & & \\ & 0 & & & & & & \\ & & 0 & & & & & \\ & & & 0 & & & & \\ & & & & 0 & & & \\ & & & & & 0 & & \\ & & & & & & 0 & 1-i \\ & & & & & & i-1 & 0 \end{pmatrix},$$

i.e.

$$\big[\hat{U}', \hat{V}'\big]_{67} = 1 - i, \qquad \big[\hat{U}', \hat{V}'\big]_{76} = i - 1.$$

This matrix is nonzero, demonstrating explicit non-commutation.

11 Symbolic Commutator Formula and Implications for Sectoral Hamiltonian Design

The counterexample in the previous section produced a commutator localized on the computational basis states $|110\rangle$ and $|111\rangle$. We now give a compact symbolic representation of this commutator and discuss design implications.

11.1 Symbolic commutator

Let

$$\hat{D}(\varphi_1, \varphi_2) = \operatorname{diag}\big(1, 1, 1, 1, 1, 1, e^{i\varphi_1}, e^{i\varphi_2}\big)$$

and retain $\hat{G}_{23}^{CNOT} = \begin{pmatrix} \text{CNOT} & 0 \\ 0 & \text{CNOT} \end{pmatrix}$ in the computational lexicographic ordering $\{|000\rangle, \ldots, |111\rangle\}$. Define

$$\hat{U}' = \hat{D}(\varphi_1, \varphi_2)\, \hat{G}_{23}^{CNOT}, \qquad \hat{V}' = \hat{G}_{23}^{CNOT}\, \hat{D}(\varphi_1, \varphi_2).$$

The commutator $\mathcal{C} := [\hat{U}', \hat{V}']$ is zero everywhere except on the two matrix elements linking $|110\rangle$ and $|111\rangle$. An operator form that captures this localization is

$$\mathcal{C} \;=\; \big(e^{i\varphi_1} - e^{i\varphi_2}\big)\,\big(\;|110\rangle\langle 111| \;-\; |111\rangle\langle 110|\;\big). \tag{44}$$

Equivalently, in components,

$$\langle 110|\mathcal{C}|111\rangle = e^{i\varphi_1} - e^{i\varphi_2}, \qquad \langle 111|\mathcal{C}|110\rangle = -(e^{i\varphi_1} - e^{i\varphi_2}),$$

all other matrix elements vanish.

11.2 Implications for sectoral Hamiltonian design

Equation (44) is compact but carries several concrete design lessons for sector-gate AGI architectures:

1. **Phase alignment controls commutativity.** If $\varphi_1 = \varphi_2$ then $\mathcal{C} = \mathbf{0}$ and the two composite orderings commute. Thus one can enforce (or exploit) commuting sectoral dynamics simply by aligning control phases on the basis states that will be permuted by downstream gates.

2. **Localized non-commutation $\Rightarrow$ localizable dynamics.** The commutator is supported only on the subspace spanned by $\{|110\rangle, |111\rangle\}$. This localization implies that non-commuting effects can be confined to small subsystems (useful for fault isolation and interpretability): design Hamiltonians so that undesired non-commutation is localized and controllable.

3. **Designing effective Hamiltonians via BCH expansions.** For small phase differences one may treat the ordering difference perturbatively. Using Baker–Campbell–Hausdorff,

$$\log\left(e^A e^B\right) = A + B + \tfrac{1}{2}[A, B] + \tfrac{1}{12}[A, [A, B]] - \tfrac{1}{12}[B, [A, B]] + \cdots,$$

shows that $[A, B]$ (and higher commutators) appear in the effective generator. In sectoral design, this means that even modest localized phase mismatches generate higher-order corrections to the effective Hamiltonian that can alter long-time dynamics—either harmful (instability, drift) or useful (richer expressivity).

4. **Control and learning knobs.** The phase parameters φ_1, φ_2 are natural control knobs: a learning rule may actively tune them to trade off between (i) commuting, stable information routing and (ii) non-commuting, expressive dynamics that create new eigenstate pathways. When embedding gates into a trainable engine, include regularization terms penalizing large $|e^{i\varphi_1} - e^{i\varphi_2}|$ unless deliberate non-commutativity is desired.

5. **Spectral and stability considerations.** Nonzero localized commutators change spectral properties of the total Hamiltonian via interaction-picture effects. Design constraints should ensure bounded operator norms of commutators to avoid amplification of noise or runaway spectral shifts (practical bounds follow from $\|\mathcal{C}\| \leq 2|e^{i\varphi_1} - e^{i\varphi_2}|$).

6. **Symmetry and conservation design.** If certain sectoral symmetries (e.g., parity, excitation number) must be preserved, impose those symmetries as constraints on the diagonal phases and inter-sector potentials V_{kl} so that commutators with symmetry generators vanish.

7. **Use-cases for engineered non-commutation.** Non-commuting sectoral modules are not always a bug: they can implement conditional plasticity, generate entangling gates useful for memory consolidation, or produce controllable interference patterns for decision making. Design patterns: (a) local phase ramps to temporally gate non-commuting interactions; (b) phase-coded memory tags where different memories intentionally produce distinct φ-profiles.

12 Sector-Gate Formalism and Quantum Eigenstate Dynamics for Artificial General Intelligence Synthesis

To situate our construction in the broader context, we note that the idea of eigenstate propagation across modular partitions was first formalized in the *Quantum Eigenstate Dynamics for Artificial General Intelligence Synthesis* framework [1]. There, the introduction of a *sector-gate* provided a minimal mathematical object capable of mediating dynamics between distinct subspaces while preserving unitary consistency.

Formally, a sector-gate can be expressed as an operator

$$\hat{S}_{kl}(\theta) \;=\; \exp\bigl(-i\,\theta\,\hat{P}_k \otimes \hat{Q}_l\bigr), \tag{45}$$

where $\hat{P}_k$ and $\hat{Q}_l$ are projectors (or Hamiltonian fragments) selecting the k-th and l-th submodules. The parameter θ controls the strength of the cross-sector coupling. In the tensor factorization picture developed above, $\hat{S}_{kl}(\theta)$ acts as a tunable cross-link between otherwise factorized subsystems.

The present work expands on this principle by embedding sector-gates directly into multi-qubit gate sequences such as $\hat{G}_{12}^{CPHASE}$ and $\hat{G}_{23}^{CNOT}$. This embedding makes explicit how eigenstates propagate across perception $\to$ reasoning $\to$ memory chains, and clarifies the commutator structure that arises when distinct sector-gates act on overlapping support. Hence we believe, such is the nature of AGI, and since we advance our understanding, we are able to build AGI.

Connection to sectoral Hamiltonians

In the Hamiltonian picture, the presence of $\hat{S}_{kl}(\theta)$ induces an effective term

$$H_{\text{eff}} \;\supset\; \theta\,\hat{P}_k \otimes \hat{Q}_l, \tag{46}$$

which can be tuned to enforce commuting or non-commuting dynamics depending on whether the associated projectors overlap on the swapped basis states. This aligns directly with the symbolic commutator structure in Eq. (44), showing how sector-gates provide a natural mechanism to design, control, or suppress cross-sector eigenstate transport.

12.1 The practical side, designing AGI

How can we make use of our newly defined principle? When designing sectoral Hamiltonians and gate embeddings, check:

- Are phases on basis states that will be permuted by downstream gates equal? (If yes, ordering safe.)
- Is the commutator norm $\|\mathcal{C}\|$ bounded to acceptable levels for stability?
- Can the nonzero commutator be used beneficially (expressivity) or should it be regularized away (robust routing)?
- Does the BCH expansion induce undesirable long-time terms? If so, compensate with engineered V_{kl} couplings.

These principles make the algebraic observation of the counterexample actionable in the engineering of quantum sector-gate AGI systems.

12.2 Remarks on matrix-phase bookkeeping

Careful indexing and consistent basis ordering is required when embedding 2×2 and 4×4 gates into larger tensor-product spaces. The explicit 8×8 matrices above are given in the computational lexicographic ordering $|q_1q_2q_3\rangle$ with q_1 the most-significant bit. When implementing alternative conventions (e.g., little-endian ordering) the same block and diagonal structures hold but must be permuted accordingly.

13 Conclusion

We formalize AGI as a **sector-gate architecture**, where cognition emerges from the interplay of sectoral Hamiltonians and interconnecting gate operators. Intelligence is therefore both the tensor product structure of $\mathcal{H}_{AGI}$ and the dynamical recursion of $\hat{G}_{kl}$ across it. We also defined a potential new design-principle of computing. This establishes a mathematically rigorous framework for an AGI engine built from sectors and gates, extending beyond binary computation into quantum self-organization.

References

[1] Peter De Ceuster Quantum Eigenstate Dynamics for Artificial General Intelligence Synthesis. Zenodo, 2024. doi:10.5281/zenodo.16748240.