A Fiber-Integration Extension of Maxwell's Equations on the Boundary of a Holographic Bulk inspired by Hertog

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Abstract

Standard quantum field theory typically models the electromagnetic vacuum on a fixed four-manifold with no additional global topological sectors. Hertog and Hawking in recent years made efforts to develop alternative theory. We consider a relaxation motivated by holographic/bulk-boundary ideas: observed 4D spacetime $\mathcal{X}$ is the base of a fibration $\pi : \mathcal{Y} \to \mathcal{X}$ with compact k-dimensional fiber Σ. Instead of invoking a derived pushforward directly at the level of differential forms, we define a concrete *transgression current* on $\mathcal{X}$ by *integration along the fiber*: a closed bulk $(3+k)$-form $\widetilde{J}$ induces a boundary 3-form

$$J_s := \pi_!(\widetilde{J}) \in \Omega^3(\mathcal{X}),$$

which is automatically conserved ($dJ_s = 0$) under mild geometric hypotheses. The effective Maxwell system becomes

$$dF = 0, \qquad d \star F = J_{\text{matter}} + J_s,$$

equivalently $d(\star F - J_s) = J_{\text{matter}}$. We give a gauge-invariant action principle, clarify when a Proca mass term is excluded, and outline (as clearly labelled speculation) how such a cohomological source could be constrained experimentally via phase-sensitive cavity or interferometric measurements.

Contents

1 Introduction: The Geometric Deficit

The Standard Model represents a triumph of reductionism, but it commonly treats the vacuum as topologically trivial for practical calculations. In contrast, many bulk/boundary frameworks suggest that observed physics on a lower-dimensional manifold $\mathcal{X}$ may emerge from a structured total space $\mathcal{Y}$. This paper explores a minimal, mathematically well-typed way for bulk topology to induce a *conserved* effective source for boundary electromagnetism.

We will be strict about two requirements:

(i) **Gauge symmetry is preserved** (no explicit $U(1)$ breaking).

(ii) **The modification is formulated in standard differential-geometric terms** on $\mathcal{X}$ (so it can be confronted with data).

1.1 The Axiomatic Basis

Axiom 1 (The Boundary/Fibration Axiom)**.** *The observed universe is modeled as a 4-dimensional Lorentzian manifold $\mathcal{X}$. There exists a smooth fibration $\pi : \mathcal{Y} \to \mathcal{X}$ whose fiber Σ is a compact, oriented k-manifold without boundary.*

Axiom 2 (Global Flux Accounting)**.** *Electromagnetic flux is globally accounted for in $\mathcal{Y}$. Apparent "extra" effective sources on $\mathcal{X}$ may arise from topological sectors on $\mathcal{Y}$ projected to $\mathcal{X}$.*

Axiom 3 (Cohomological Vacuum Sectors)**.** *The bulk admits closed differential forms representing non-trivial cohomology classes (equivalently: non-trivial vacuum sectors). We will work in the de Rham model; more categorical/topos language can be viewed as an organizing framework for these sectors, but is not required for the core equations.*

2 Theoretical Framework: Fiber Integration and a Conserved Source

2.1 From bulk forms to a boundary 3-form

Definition 2.1 (Integration along the fiber)**.** *Let $\pi : \mathcal{Y} \to \mathcal{X}$ be a smooth fibration with compact oriented fiber Σ of dimension k and no boundary. There is a standard pushforward operation on forms (sometimes called "fiber integration")*

$$\pi_! : \Omega^{p+k}(\mathcal{Y}) \to \Omega^p(\mathcal{X}),$$

defined by integrating a form over each fiber. (Locally, in a trivialization $\mathcal{Y} \cong \mathcal{X} \times \Sigma$, it reduces to $\pi_!(\alpha)(x) = \int_\Sigma \alpha(x, \sigma)$.)

Lemma 2.2 (Fiberwise Stokes identity)**.** *If Σ is compact and $\partial\Sigma = \varnothing$, then*

$$d\,\pi_!(\alpha) = \pi_!(d\alpha) \qquad \text{for all } \alpha \in \Omega^\bullet(\mathcal{Y}).$$

Proof. This is the standard compatibility of fiber integration with exterior differentiation, following from Stokes' theorem on each fiber and $\partial\Sigma = \varnothing$. $\square$

Definition 2.3 (Bulk-to-boundary transgression current). *Let $\widetilde{J} \in \Omega^{3+k}(\mathcal{Y})$ be a* closed *bulk form:*

$$d\widetilde{J} = 0.$$

Define the induced boundary 3-form

$$J_s := \pi_!(\widetilde{J}) \in \Omega^3(\mathcal{X}).$$

Theorem 2.4 (Conservation of the transgression current). *If $d\widetilde{J} = 0$ and $\partial\Sigma = \varnothing$, then*

$$dJ_s = 0.$$

Proof. By the fiberwise Stokes identity,

$$dJ_s = d\,\pi_!(\widetilde{J}) = \pi_!(d\widetilde{J}) = \pi_!(0) = 0.$$

$\square$

Remark 2.5 (Relation to derived pushforward). *If one prefers sheaf-theoretic language, the above construction corresponds to taking the de Rham representative of a cohomology class and pushing it forward via the appropriate functor on cohomology. In this paper we keep the de Rham/fiber-integration model explicit so that every object lives in a single category (smooth forms on manifolds).*

2.2 Extended Maxwell system with matter included

Let $A \in \Omega^1(\mathcal{X})$ be a $U(1)$ gauge potential and $F = dA \in \Omega^2(\mathcal{X})$ its curvature. Let $J_{\text{matter}} \in \Omega^3(\mathcal{X})$ denote the ordinary matter current 3-form (so that $dJ_{\text{matter}} = 0$ expresses charge conservation).

Definition 2.6 (Extended Maxwell equations). *The proposed bulk-induced extension is*

$$dF = 0, \qquad d \star F = J_{\text{matter}} + J_s. \tag{1}$$

Equivalently,

$$d(\star F - J_s) = J_{\text{matter}}.$$

Remark 2.7 (Consistency with charge conservation). *Applying d to the second equation gives $0 = dJ_{\text{matter}} + dJ_s$. Hence if $dJ_s = 0$ (proved above), the usual conservation law $dJ_{\text{matter}} = 0$ is unchanged.*

2.3 Action principle and gauge invariance

Definition 2.8 (Gauge-invariant action on $\mathcal{X}$). *Consider the action functional*

$$S[A] = -\frac{1}{2}\int_{\mathcal{X}} F \wedge \star F \;+\; \int_{\mathcal{X}} A \wedge (J_{\text{matter}} + J_s), \tag{2}$$

with suitable boundary conditions (e.g. compact support variations or fields decaying at infinity).

Lemma 2.9 (Euler–Lagrange equations). *Varying* (2) *with respect to A yields* (1).

Proof. Standard variational calculus gives $\delta F = d(\delta A)$ and hence $\delta \int F \wedge \star F = 2\int d(\delta A) \wedge \star F$. Integrating by parts and dropping boundary terms under standard assumptions yields $\int \delta A \wedge d \star F$. Adding the source term gives $d \star F = J_{\text{matter}} + J_s$. $\square$

Theorem 2.10 (Gauge invariance under closed currents)**.** *Under a gauge transformation $A \mapsto A + d\lambda$,*

$$\delta S = \int_{\mathcal{X}} d\lambda \wedge (J_{\text{matter}} + J_s) = -\int_{\mathcal{X}} \lambda \wedge d(J_{\text{matter}} + J_s),$$

up to boundary terms. Therefore, if $dJ_{\text{matter}} = 0$ and $dJ_s = 0$ (and boundary terms vanish), the action is gauge invariant.

2.4 Masslessness: seperating facts from fiction

Theorem 2.11 (Sideremark: No Proca mass term in the effective boundary action (under standard locality assumptions))**.** *The action* (2) *contains no term of the form $\frac{m^2}{2}\int_{\mathcal{X}} A \wedge \star A$, hence no explicit Proca mass is introduced at the boundary level. Moreover, any* local *and* Lorentz-invariant *deformation that preserves $U(1)$ gauge invariance cannot generate a Proca mass term.*

Proof. A Proca mass term is not gauge invariant under $A \mapsto A + d\lambda$ and is therefore excluded by exact $U(1)$ invariance (assuming no additional Stueckelberg fields are introduced). Since (2) is gauge invariant when the currents are closed, it forbids such a local mass term in the boundary action. □

Remark 2.12. *Integrating out bulk degrees of freedom can, in principle, generate* nonlocal *but gauge-invariant terms (e.g. involving F and inverse differential operators). Such terms do not constitute a Proca mass, but they can modify propagation/dispersion. CERN should take note, how any phenomenology must specify the form and scale of these corrections for any future theory to be considered valid.*

3 Experimental Optional Modelling Layer: Rough Interfaces, road to nowhere or breakthrough?

The construction above is complete without stochastic analysis. If one wishes to model fluctuations in how bulk sectors populate $\widetilde{J}$ (or how π varies effectively), one may introduce a stochastic dynamics for parameters that control $\widetilde{J}$. Tools like regularity structures are *potentially* relevant only after a specific singular SPDE is derived from a microscopic model. In this work we merely note this as a possible future direction and do not rely on it for the main theorems since such would be indeed highly experimental. However if anyone wishes to proceed with such angle, Hairer has accomplished a good deal of work related to these stochastic sciences.

4 Phenomenological Theorum (Hypotheses)

4.1 Fusion plasmas as a constraint channel

Tokamak "anomalous transport" is usually attributed to turbulence and micro-instabilities. In the present framework, a *testable* approach would be:

(a) write J_s in terms of a small number of measurable parameters (an ansatz),

(b) derive a modified energy or particle balance law,

(c) compare with existing transport scalings to set bounds on the amplitude of J_s.

Without these proper steps, fusion remains a motivating analogy rather than an established application. We desire prolonged fusion, hence such experimental direction might be productive for future researchers to study, however success is not guaranteed.

4.2 Sideremark: Solar system perturbations

Any claim that a bulk defect mimics a "dark focus" must be confronted with existing planetary ephemerides constraints, wide-field surveys, and dynamical stability analyses. In our work we treat this only as a speculative interpretation, a mere reference point and not as a derived result. It is important the holographic topic is approached with caution, not to cross into pseudosciences.

5 Experimental Handle: Phase-sensitive cavity bounds

Because J_s couples through $\int A \wedge J_s$, it is natural to search for small, coherent phase effects rather than dissipative losses. A productive next step is to choose an explicit family of closed forms $\widetilde{J}$ (or their cohomology classes), compute the induced J_s, and derive a concrete observable such as a phase shift or resonance-like response in a cavity geometry. This is not a completed prediction, rather an early outline. Again, nonetheless, such direction might be of interest to future researchers.

6 Discussion in experimental physics

The core contribution of this abstract is a minimal, well-typed mechanism for inducing a conserved 3-form source on $\mathcal{X}$ from bulk topology, without breaking $U(1)$ gauge invariance:

$$J_s = \pi_!(\widetilde{J}), \qquad d\widetilde{J} = 0 \;\Rightarrow\; dJ_s = 0,$$

and the consistent Maxwell system

$$dF = 0, \qquad d \star F = J_{\text{matter}} + J_s.$$

More ambitious interpretive layers (topos logic, rough-interface SPDEs, cosmological anomalies) should be built on top of this foundation only after specifying models and scales. It seems realistic to build further upon holographic theory.

7 Conclusion

We presented a gauge-invariant extension of Maxwell theory sourced by a conserved, cohomologically-induced 3-form J_s obtained via fiber integration from a closed bulk form. We have shown holographic theory has potential. The framework is mathematically explicit, preserves the usual charge conservation identity, and avoids introducing an explicit Proca mass term. Future work must specify $\widetilde{J}$ (or its parameterization) to obtain falsifiable predictions and experimental bounds.

Sources of inspiration

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