Topological Unification of Axion Electrodynamics and Higher-Dimensional Photon–Higgs Continuity

Peter De Ceuster

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Abstract

We propose a unified extension of Maxwell's equations that bridges axion electrodynamics with higher-dimensional geometric constraints, offering a singular theoretical framework to probe sub-atomic spacetime structures across both ultra-high-energy particle colliders and low-energy precision quantum-optical facilities. By integrating the topological θ-term of pseudo-scalar axion fields with a hidden "soul" current J_s—derived from a $(4+k)$-dimensional bulk Higgs–photon coupling—we formulate a generalized electrodynamic continuity law of the form $d(*F - \theta F - J_s) = 0$. In ordinary low-energy four-dimensional spacetime ($k = 0$), the extra-dimensional current vanishes identically ($J_s = 0$), smoothly recovering standard axion electrodynamics and classical Maxwellian behavior at macroscopic scales. At ultra-small distance scales (Δx) approaching the extra-dimensional compactification boundary Λ, however, the geometric interplay between axion–photon oscillations and the conserved topological soul-charge Q_{soul} becomes dynamically active, with deviations from classical electrodynamics scaling as Λ^{-2k}. We demonstrate that this combined topological framework yields complementary experimental signatures across opposite energy frontiers. In the high-energy regime, future collider facilities such as the Future Circular Collider (FCC) at CERN can probe high momentum transfers (Δp) near Λ to simultaneously search for dark-matter axion candidates and extra spatial dimensions via precision measurements of anomalous Higgs–photon vertices and electroweak gauge extensions. Conversely, in the low-energy tabletop regime, the cohomology obstruction class $\omega(\eta)$ encoded by J_s induces a measurable degradation in quantum photon interference visibility, $1-V \sim \|\omega(\eta)\|^2 \Lambda^{-2k}$, alongside modified photon–axion conversion rates in intense magnetic fields. Furthermore, slowing and trapping light pulses inside ultra-cold atomic Bose–Einstein condensates (BEC) via electromagnetically induced transparency (EIT) provides a coherent light–matter storage medium capable of amplifying interaction times and imprinting topological phase and spin excitations without information loss.

1 Unified Topological Continuity Law

Classical four-dimensional electrodynamics is governed by the homogeneous and inhomogeneous Maxwell equations for the field-strength 2-form $F = dA$ on 4D spacetime X:

$$dF = 0, \qquad d{*}F = 0, \tag{1}$$

which reflect an unbroken $U(1)$ local gauge symmetry with massless, Higgs-decoupled photons. While Yang–Mills theories generalize this gauge principle to non-commutative $SU(2)$ and $SU(3)$ symmetry groups within the Standard Model, topological extensions modify the effective electromagnetic current directly.

To unify pseudo-scalar axion electrodynamics with higher-dimensional Higgs–photon interactions, let $\pi : Y \to X$ denote the projection from a $(4+k)$-dimensional bulk manifold Y onto 4D spacetime X. Following the sheaf-theoretic formulation [1, 3]:

- Let P denote the photon sheaf on Y pulled back from X.
- Let H denote the bulk Higgs sheaf supported on Y.
- Let $\eta : \pi^*P \otimes H \to \mathcal{O}_Y[l]$ be a nontrivial morphism in the derived category encoding the higher-dimensional Higgs–photon coupling.

The induced higher-dimensional "soul" current on X is defined via the derived pushforward as a differential 3-form:

$$J_s = R^\bullet \pi_* \big[\eta(\pi^*P \otimes H)\big] \in \Omega^3(X). \tag{2}$$

Incorporating the axion topological coupling θF, where θ represents the pseudo-scalar axion field normalized by its coupling constant, we postulate the unified field equations:

$$dF = 0, \qquad d(*F - \theta F - J_s) = 0. \tag{3}$$

In ordinary four-dimensional spacetime ($k = 0$), the bulk Higgs sheaf H has no support off Y, forcing $\pi_*\eta(\pi^*P \otimes H) = 0$ and $J_s = 0$. Equation (3) then reduces directly to standard axion electrodynamics $d(*F - \theta F) = 0$, and in the absence of background axion gradients, to classical Maxwell theory [2]. In the full $(4 + k)$-dimensional theory, integrating over a spatial 3-cycle $\Sigma^3 \subset X$ yields a generalized conserved topological charge:

$$Q_{\text{soul}} = \int_{\Sigma^3} (*F - \theta F - J_s) \implies d(*F - \theta F - J_s) = 0. \tag{4}$$

2 Dual Experimental Regimes: Collider and Quantum-Optical Probes

The unified continuity law $d(*F - \theta F - J_s) = 0$ establishes a direct bridge between ultra-high-energy particle physics and ultra-cold tabletop quantum optics.

2.1 High-Energy Collider Regime (CERN and Future Colliders)

By the quantum uncertainty relation linking spatial resolution Δx to momentum transfer Δp, probing substructure below 10^{-19} m and approaching the compactification scale Λ requires ultra-high collision energies:

- **Higgs–Photon Vertex Precision:** Because J_s originates from the bulk morphism $\eta(\pi^*P \otimes H)$, high-energy collisions at CERN and proposed facilities such as the Future Circular Collider (FCC) can test anomalous Higgs–photon couplings and electroweak unification thresholds sensitive to Λ^{-2k}.
- **Dark Matter and Extra-Dimensional Signatures:** High-luminosity collider environments probe the simultaneous production of weakly interacting axion-like particles (θ) and Kaluza–Klein or sheaf-cohomological excitations associated with the k compactified dimensions.

2.2 Low-Energy Tabletop Quantum-Optical Regime

Complementing collider searches, low-energy precision optical experiments test the topological imprint of both θF and J_s with extreme phase sensitivity:

- **Interferometric Visibility Degradation:** The current J_s encodes a cohomology obstruction class $\omega(\eta)$ reinterpreted as a 3-form on X. A non-vanishing obstruction class produces a quantifiable loss of perfect quantum interference visibility:

$$1 - V \sim \|\omega(\eta)\|^2 \Lambda^{-2k}, \tag{5}$$

  which can be bounded or detected in high-finesse optical interferometers alongside magnetic birefringence from photon–axion conversion.

- **Slow and Stopped Light in Bose–Einstein Condensates (BEC):** Utilizing electromagnetically induced transparency (EIT) in clouds of ultra-cold atoms cooled to nanokelvin temperatures, coherent light pulses can be decelerated by orders of magnitude or temporarily halted as collective atomic spin excitations. Mapping the unified field $*F - \theta F - J_s$ into a BEC light-storage medium drastically extends the effective interaction time, enabling sensitive quantum-memory tests of topological phase drifts and light–matter conversion anomalies.

Experimental Regime	**Physical Mechanism**	**Primary Observable**
High-Energy Colliders (CERN/FCC)	High-Δp probing near scale Λ	Anomalous $H\gamma$ vertex & ax
High-Field Optical Interferometry	Cohomology obstruction $\omega(\eta)$ & θF	Visibility loss $1 - V \sim \|\omega(\eta)$
Ultra-Cold Atomic BEC + EIT	Slow/stopped light spin-wave storage	Phase-coherent light–matt

Table 1: Summary of complementary high-energy and low-energy experimental pathways probing the unified continuity law $d(*F - \theta F - J_s) = 0$.

References

[1] D. S. Freed, *Dirac charge quantization and generalized differential cohomology*, Surveys in Differential Geometry **7**, 129–194 (2000).

[2] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1995).

[3] E. Witten, *Geometric Langlands and the equations of Nahm and Bogomolny*, arXiv:1602.09021 [hep-th] (2016).